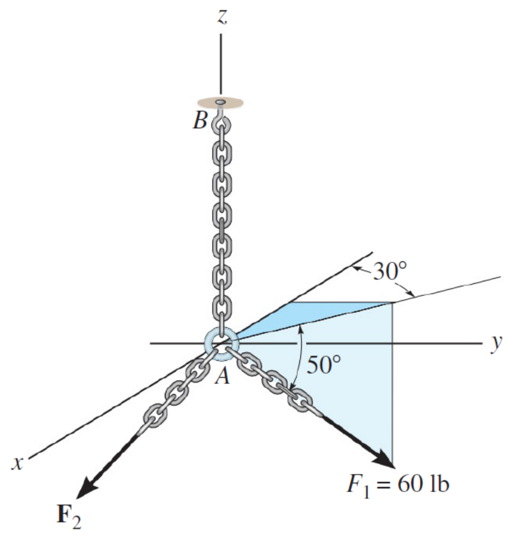
Problem 2-78

The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



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Solution

Write each of the forces in component form.

$$\mathbf{F}_1 = 60\,\langle -\cos 50^\circ \cos 30^\circ, \cos 50^\circ \sin 30^\circ, -\sin 50^\circ\rangle \text{ lb}$$

$$\mathbf{F}_2 = F_2\,\langle \cos\alpha_2, \cos\beta_2, \cos\gamma_2\rangle \text{ lb}$$

$$\mathbf{F}_R = \langle 0, 0, -100\rangle \text{ lb}$$

Add them together to get the resultant force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\langle 0, 0, -100\rangle \text{ lb} = \langle -60\cos 50^\circ \cos 30^\circ + F_2\cos\alpha_2, 60\cos 50^\circ \sin 30^\circ + F_2\cos\beta_2, -60\sin 50^\circ + F_2\cos\gamma_2\rangle \text{ lb}$$

Match the components to get a system of equations.

$$0 = -60\cos 50^\circ \cos 30^\circ + F_2\cos\alpha_2$$

$$0 = 60\cos 50^\circ \sin 30^\circ + F_2\cos\beta_2$$

$$-100 = -60\sin 50^\circ + F_2\cos\gamma_2$$

Solve for the terms with F_2.

$$F_2\cos\alpha_2 = 60\cos 50^\circ \cos 30^\circ \tag{1}$$

$$F_2\cos\beta_2 = -60\cos 50^\circ \sin 30^\circ \tag{2}$$

$$F_2\cos\gamma_2 = -100 + 60\sin 50^\circ \tag{3}$$

Square both sides of each equation and add the respective sides together to get F_2.

$$F_2^2(\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2) = (60\cos 50^\circ \cos 30^\circ)^2 + (-60\cos 50^\circ \sin 30^\circ)^2 + (-100 + 60\sin 50^\circ)^2$$

$$F_2^2(1) = (60\cos 50^\circ \cos 30^\circ)^2 + (-60\cos 50^\circ \sin 30^\circ)^2 + (-100 + 60\sin 50^\circ)^2$$

$$F_2 = \sqrt{(60\cos 50^\circ \cos 30^\circ)^2 + (-60\cos 50^\circ \sin 30^\circ)^2 + (-100 + 60\sin 50^\circ)^2}$$

$$F_2 \approx 66.4 \text{ lb}$$

Plug this value for F_2 back into equations (1), (2), and (3) to determine α_2, β_2, and γ_2.

$$\begin{cases} \cos\alpha_2 = \dfrac{60\cos 50^\circ \cos 30^\circ}{F_2} \\ \cos\beta_2 = \dfrac{-60\cos 50^\circ \sin 30^\circ}{F_2} \\ \cos\gamma_2 = \dfrac{-100 + 60\sin 50^\circ}{F_2} \end{cases} \quad \rightarrow \quad \begin{cases} \alpha_2 \approx 59.8^\circ \\ \beta_2 \approx 107^\circ \\ \gamma_2 \approx 144^\circ \end{cases}$$